**Homework 3**

**Instructions:** Do as many of the problems as you like, but make sure to complete at least **three**. Then the whole class will create a solution jointly (write below, or create a separate file; either is OK). One problem can have multiple solutions, so if your solution is different than the one already posted and you’d like to share yours with others, feel free to add yours. Make sure to separate different solutions to minimize confusion.

1. Thinking like an instructor: Make up a solvable Diophantine equation. Explain what process you used and which results you relied on during the process.
   * I first created two numbers out of prime factors so I would easily know their gcd. I chose and so their gcd is 6.
   * From there I used the result that any linear combination of the two numbers will result in a number which is a multiple of their gcd. Thus I could set their linear combination equal to any multiple of 6 and I chose 12.
2. Use Euclidean algorithm to find the gcd of the following pairs:   
   a. 123, 456

* gcd(123, 456) = gcd(123, 87)
  + - = gcd(87, 36)
    - = gcd(36, 51)
    - = gcd(36, 15)
    - = gcd(15, 6)
    - = gcd(6, 3)
    - = gcd(3, 0)
    - = 3

b. 1234, 567

* gcd(1234, 567) = gcd(567, 100)
  + - = gcd(100, 67)
    - = gcd(67, 33)
    - = gcd(33, 1)
    - = 1

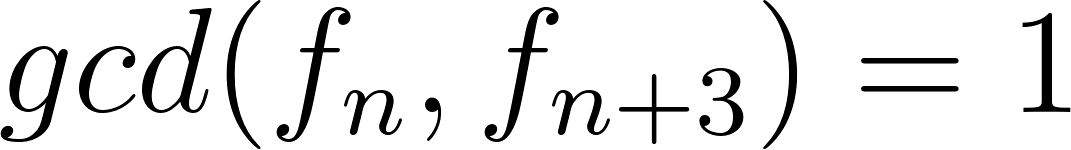
1. What are the possible values of? If you guess a possible value, make sure that it occurs.

* so, by calculating remainders,
* .
* if and only if is not a multiple of 3.
  + This is because if is not a multiple of 3 then there exists such that which using the as a linear combination means that .
  + Conversely, if is a multiple of 3 then and will share a factor of at least 3 and their will be at least 3, and thus not 1.
* Thus, should all work.
* Checking these with my calculator yields:
* which are all true.

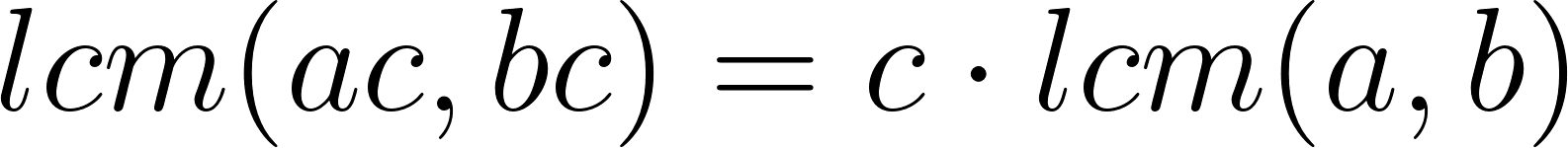
1. a. Find an integer solution to the equation.

* using the extended euclidean algorithm

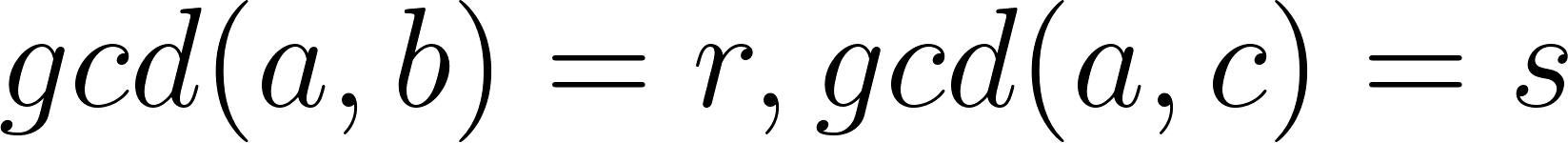
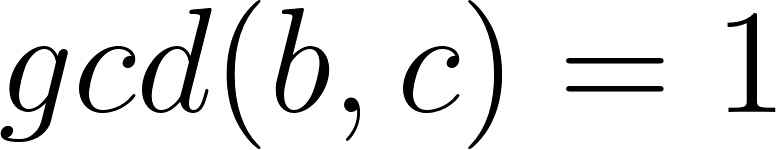
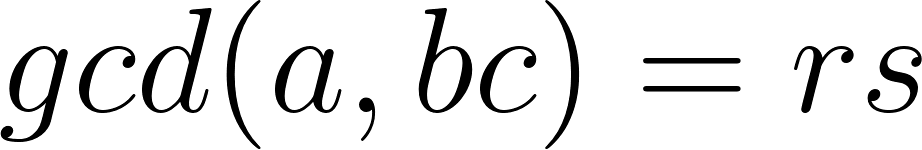
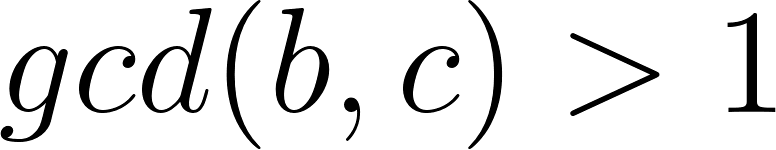
b. Find a second solution with *x>*0.

1. Show that [](https://www.codecogs.com/eqnedit.php?latex=gcd(f_n%2C%20f_%7Bn%2B3%7D)%3D1#0) or 2.

* Thus, by substituting this into the above equation and calculating remainders,
  + - * ...?

1. Prove that if *c>*0, then [](https://www.codecogs.com/eqnedit.php?latex=lcm(ac%2Cbc)%3Dc%5Ccdot%20lcm(a%2Cb)#0).

* By definition of a common multiple, for some integers .
* Multiplying each part of this equation by we obtain
* By definition of a common multiple, is a common multiple of and .
* Therefore . (1)
* By definition of a common multiple, for some integers .
* With these coefficients we can see that is multiplied by a common multiple of and .
* Therefore . (2)
* Combining (1) and (2) gives us that .

1. Show that if [](https://www.codecogs.com/eqnedit.php?latex=gcd(a%2Cb)%3Dr%20%2C%20gcd(a%2Cc)%3Ds#0) and [](https://www.codecogs.com/eqnedit.php?latex=gcd(b%2Cc)%3D1#0), then [](https://www.codecogs.com/eqnedit.php?latex=gcd(a%2Cbc)%3Drs#0). Give an example to show that this need not be true if [](https://www.codecogs.com/eqnedit.php?latex=gcd(b%2Cc)%3E1#0).

* I understand this to be true from a more set theory approach (which in my head is always visualized as Euler diagrams). If we consider each number as a set of its prime factors, is then finding an intersection, and multiplying numbers together is the addition of their sets. For disjoint sets ( and since , also and since these are subsets of and ) their set addition is equivalent to their union which is equivalent to their number multiplication.
* , ,
* But I have not gotten through a more number theory approach.
* Example if :

1. Write a code to implement the Euclidean algorithm to find gcd.

* def gcd(a, b):
  + if a<0 or b<0:
    - return gcd(abs(a), abs(b))
  + if a==1 or b==1:
    - return 1
  + if a==0 and b==0:
    - raise Exception("gcd(0, 0) is not defined")
  + if a==0 or b==0:
    - return max(a, b)
  + if a<b:
    - return gcd(a, b%a)
  + if b<a:
    - return gcd(b, a%b)
  + return a